MULTIMEDIA UNIVERSITY

STUDENT ID NO

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 3, 2018/2019**

## PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

29 May 2019
2.30 p.m. – 4.30 p.m.
(2 Hours)

---

**INSTRUCTIONS TO STUDENT**

1. This Question paper consists of 2 pages excluding the cover page and appendix.
2. Answer **all FIVE** questions. Each question carries equal marks and the distribution of the marks is given.
3. Write all your answers in the Answer Booklet provided.

**Question 1 [10 marks]**

a. Find the following limits.

i. $\lim_{x\to -1}\dfrac{2x^2-x-3}{3x^2+8x+5}$ (2 marks)

ii. $\lim_{x\to 0}\dfrac{x+\sin 4x}{3x}$ (2 marks)

iii. $\lim_{x\to 0}\dfrac{\sqrt{4+x}-\sqrt{4-x}}{x}$ (3 marks)

b. Find all values of $a$ such that the function below is continuous everywhere.

$$f(x)=\begin{cases} x^2+2x, & x<a \\ -1, & x\geq a \end{cases}$$ (3 marks)

**Question 2 [10 marks]**

a. i. Find the equation of the tangent line to $y=4\sqrt{2x}-6e^{2-x}$ at $x=2$. (3 marks)

ii. Differentiate $f(x)=\ln(\sin x)-(x^4-3x)^{10}$. (2 marks)

b. Evaluate $\int_2^3 \dfrac{5x+4}{(x+2)^2(x-1)}dx$. (5 marks)

**Question 3 [10 marks]**

a. Given $f(x)=x^3-9x^2+24x$.

i. Find the critical numbers and identify the maximum and/or minimum point(s). (3 marks)

ii. Find the $x$-coordinate(s) of inflection point(s) if any. (2 marks)

iii. Hence, sketch the graph $f(x)=x^3-9x^2+24x$ (2 marks)

**Continued...**

b. A cylindrical tank standing upright (with one circular base on the ground) has radius 20 cm. How fast does the water level in the tank drop when the water is being drained at 25 $cm^3$/sec? (3 marks)

**Question 4 [10 marks]**

a. Given a curve $x = 3y - y^2$ and the line $x + y = 3$.

i. Find the coordinates of the points of intersection of the line and the curve. (2 marks)

ii. Find the area of the region enclosed by the line and the curve. (3 marks)

b. Sketch the graph of a curve $y = 2\sqrt{x}$ and $y = x$. Shade the region enclosed by $y = 2\sqrt{x}$ and $y = x$. Then find the volume of the solid obtained by rotating the shaded region around the $x$-axis. (5 marks)

**Question 5 [10 marks]**

a. Find the general solution for the first order differential equation $y' = x^2e^y + 4x^3e^y$. (4 marks)

b. Find the unique solution for the second order differential equation

$$y'' + 12y' + 36y = 0 \text{ where } y(0) = 5 \text{ and } y'(0) = -10$$

(6 marks)

**End of Paper**

**APPENDIX**

**A. Differentiation Rules**

$\frac{d}{dx}[x^n] = nx^{n-1}$ ; $n$ is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$ ; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ ; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$ ; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$ ; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad \frac{d}{dx}[\cos x] = -\sin x$ $\quad \frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad \frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad \frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad \frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

**B. Basic Integration Formulas**

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad \int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad \int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad \int \frac{1}{x}dx = \ln|x| + C$

**Integration by-parts:** $\int u\,dv = uv - \int v\,du$

**Volume (disk)** $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

**Volume (washer)** $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$